Note: This paper has been revised. See Equation A8 and the Addendum

Use of *a Priori* Parameter Estimates in the Derivation of Spatially Consistent Parameter Sets of Rainfall-runoff Models

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It is well known that hydrologists rely much on trail-and-error process in estimating conceptual model parameters. While there has been a great deal of research into the development of automatic calibration methods, subjective expert judgment still plays a significant role in the selection of 'optimal' parameter sets. Any technique for calibrating rainfall-runoff model parameters requires many years of historical hydrometeorological data, and usually performs a single basin analysis. The quality and quantity of historical data can vary significantly for different regions, and even for different river basins in the same region. These inconsistencies can lead to non-optimal calibration results, and consequently significant and inappropriate randomness in the spatial patterns of model parameters. Therefore, an objective estimation procedure is needed that can produce spatially consistent and physically realistic parameter values. This paper investigates the possibility of using *a priori* parameter estimates to improve the calibration/estimation process. A set of physically based relationships between the Sacramento Soil Moisture Accounting model parameters and soil properties were developed to estimate *a priori* parameter values. Two tests, model parameter transferability to ungaged basins and constrained automatic calibration, were performed for a number of headwater watersheds in the Ohio river basin. The results suggest that the use of soil derived parameters can improve the spatial and physical consistency of parameter estimates while maintaining hydrological performance. Soil derived parameters provide a quantitative measure of possible differences between parameters of neighboring basins that allow one to 'rescale' calibrated parameters to ungaged watershed. Use of constrained calibration reduces inappropriate randomness in the spatial pattern of model parameters.

1. INTRODUCTION

The successful application of any rainfall-runoff model greatly depends on its parameterization. It is well known that hydrologists rely much on trail-and-error process in estimating of conceptual model parameters because their parameters generally are not directly observable [*Duan et al.*, 2001]. Even if model parameters are related to observable physical properties (e.g., parameters of so-called 'physically-based' models), some fine tuning or calibration of *a priori* parameters would still be required because basin-scale heterogeneities of physical properties and data uncertainties could significantly affect on an estimation process [*Carpenter et al.*, 2001]. While there has been a great deal of research into the development of automatic calibration methods (e.g., see *Rajaram and Georgakakos*, [1989]; *Sorooshian and Gupta*, [1995]; *Yapo et al.*, [1998]), subjective expert judgment still plays a significant role in the selection of 'optimal' parameter sets [*Hogue et al.*, 2000]. Existing calibration techniques tend to produce 'noisy'

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10/1029/006WS18

parameter estimates. As stated by Burnash, [1995], "This could occur because the data set was not stable, the historic sequence did not include an adequate sequence of events to exercise some of the model's characteristics, or the optimization function was not sensitive to the discrete functions associated with the proper use of particular parameters." Implementation of fully distributed models increases requirements to the calibration system to preserve a physically reasonable spatial pattern of model parameters [*Refsgaard*, 1997].

Without a systematic approach, spatial inconsistencies can enter the calibration process at several points. For example, any technique for calibrating rainfall-runoff model parameters requires many years of historical hydrometeorological data, including precipitation, temperature, streamflow discharge, etc. The quality and quantity of these data can vary significantly for different regions, and even for different river basins in the same region. In addition, it is common for each river basin to be calibrated independently. Moreover, parameters can sometimes be given values that cause the process they represent to be simulated improperly, even though the overall statistical results indicate a good fit. These inconsistencies can lead to non-optimal calibration results, and consequently significant and inappropriate randomness in the spatial patterns of model parameters. Therefore, an objective estimation procedure is needed that can produce spatially consistent and physically realistic parameter values. The procedure should be constrained so that parameter adjustment takes place within a range of values which retains conceptual consistency. This paper investigates the possibility of using *a priori* parameter estimates to improve the calibration/estimation process. Section 2 contains a brief overview of the Sacramento Soil Moisture Accounting (SAC-SMA) model that was used in analysis, and an approach by Koren et al., [2000] to generate *a priori* estimates of the SAC-SMA model parameters from soil-vegetation data. Section 3 discusses a practical procedure of estimation of soil derived SAC-SMA model parameters, and the experimental design for testing the use of these estimates in the derivation of spatially consistent parameters. Test results are presented in Section 4. Section 5 contains a summary and recommendations for future work.

2. SOIL-BASED ESTIMATES OF SAC-SMA MODEL PARAMETERS

Parameters of conceptual models such as the SAC-SMA model are usually derived from input-output data analysis using automatic or manual calibration procedures, but are not readily derived from physical basin characteristics. This deficiency restricts the application of these models (e.g., limited use in ungaged basins, high spatial resolution applications, etc.) significantly. Improvements in quality and quantity of high resolution GIS data have stimulated developments of regional relationships between basin properties and model parameters which could be used in *a priori* parameter estimation. Abdulla et al., [1996] derived empirical equations which correlate the VIC-2L LSM parameters to easily determinable basin characteristics for the GCIP Large Scale Area-Southwest. Duan et al., [1996] correlated the parameters of the Simple Water Balance (SWB) model and basin characteristics for the southeast quadrant of the US. In both cases, model parameters were calibrated for selected basins prior to the derivation of regression equations. The disadvantage of this approach is that the calibration procedure can introduce significant uncertainties in the 'optimal' parameter set, and subsequently into the regression equations because the input/output data are noisy. Recently, soil/vegetation data were explicitly used to derive physically-based analytical relationships between soil properties and conceptual model parameters. Knowles, [2000] developed such relationships for the Bay-Delta Watershed Model (BDWM). The BDWM structure is similar to the conceptual structure of the SAC-SMA model. Koren et al., [2000] developed analytical relationships for the most SAC-SMA model parameters. In this study, we adopted an approach developed by Koren et al., [2000] that uses high resolution soil and vegetation data.

2.1. SAC-SMA Model Structure and Parameters

A detailed description of SAC-SMA can be found in Burnash et al., [1973] and Burnash, [1995]. The basic design of the SAC-SMA model centers on a two layer structure: a relatively thin upper layer, and usually a much thicker lower layer which supplies moisture to meet the evapotranspiration demands. Each layer consists of tension and free water storages that interact to generate soil moisture states and five runoff components. The free water storage of the lower layer is divided into two sub-storages: the *LZFSM* which controls supplemental (fast) base flow, and the *LZFPM* which controls primary (slow) ground water flow. Partitioning of rainfall into surface runoff and infiltration is constrained by the upper layer soil moisture conditions and the percolation potential of the lower layer. No surface runoff occurs before the tension water capacity of the upper layer, *UZTWM*, is filled. After that, surface runoff generation is controlled by the content of the upper layer free water storage, *UZFWM*, and the deficiency of lower layer tension water, *LZTWM*, and free water storages. Each free water reservoir can generate runoff depending on a depletion coefficient of the reservoir, namely the *UZK* coefficient for the upper layer, and *LZSK* and *LZPK* for the lower layer supplemental and primary free water storages, respectively. The percolation rate into the lower layer, I_{perc}, is a nonlinear function of the saturation of lower layer reservoirs, W_{LZ}, and the upper layer free water reservoir, W_{UZF}:

$$I_{perc} = I_o [1 + ZPERC \cdot (1 - \frac{W_{LZ}}{LZWM})^{REXP}] \frac{W_{UZF}}{UZFWM} \quad (1)$$

where *ZPERC* is a ratio of maximum and minimum percolation rates, *REXP* is an exponent value that controls the shape of the percolation curve, *LZWM=LZTWM+LZFSM +LZFPM* is a total capacity of the lower layer, and I_o is the minimum percolation rate under fully saturated conditions in the upper and lower layers which equals the maximum rate of drainage from lower layer free water storages:

$$I_o = LZFSM \cdot LZSK + LZFPM \cdot LZPK \quad (2)$$

Percolated water into the lower layer is divided among three storages of the layer. A parameter *PFREE* is used to express the fractional split of percolated water between tension and free water storages of the lower layer.

There are five minor parameters that control impermeable area runoff and riparian evapotranspiration. Table 1 lists all SAC-SMA model parameters.

Although there are strong physical arguments to support the model [*Burnash*, 1995], 16 model parameters can not be measured. Some helpful rules were suggested for estimating of initial values of SAC-SMA parameters using hyetograph-hydrograph analysis [*Burnash*, 1995]. These initial estimates play a key role in the manual calibration procedure of the National Weather Service River Forecast System (NWS-RFS) [*Smith et al.*, this volume]. However, this procedure is based on trial-and-error approach and depends much on expert experience.

Recent developments by the University of Arizona research group [*Boyle et al.*, 2001; *Boyle et al.*, 2000; *Hogue et al.*, 2000; *Yapo et al.*, 1998; *Sorooshian and Gupta*, 1995] have significantly improved the automatic calibration process of the SAC-SMA model. However, limitations on the selection of an objective function, structural problems of the model, and uncertainties in input/output data reduce the ability of automatic calibration to obtain unique and conceptually realistic parameter estimates. On the other hand, a single basin calibration approach limits the analyses of the spatial pattern of model parameters, and can lead to inappropriate spatial randomness of calibration results.

2.2. Soil Texture and SAC-SMA Model Parameter Relationships

Koren et al., [2000] developed a physically based approach to quantify the relationships of 11 major parameters of the SAC-SMA model with soil properties (these parameters are highlighted in Table 1). As defined in Section

Table 1. SAC-SMA model parameters and their feasible ranges.

No	Parameter	Description	Ranges
1	***UZTWM***	The upper layer tension water capacity, *mm*	10-300
2	***UZFWM***	The upper layer free water capacity, *mm*	5-150
3	***UZK***	Interflow depletion rate from the upper layer free water storage, *day*$^{-1}$	0.10-0.75
4	***ZPERC***	Ratio of maximum and minimum percolation rates	5-350
5	***REXP***	Shape parameter of the percolation curve	1-5
6	***LZTWM***	The lower layer tension water capacity, *mm*	10-500
7	***LZFSM***	The lower layer supplemental free water capacity, *mm*	5-400
8	***LZFPM***	The lower layer primary free water capacity, *mm*	10-1000
9	***LZSK***	Depletion rate of the lower layer supplemental free water storage, *day*$^{-1}$	0.01-0.35
10	***LZPK***	Depletion rate of the lower layer primary free water storage, *day*$^{-1}$	0.001-0.05
11	***PFREE***	Percolation fraction that goes directly to the lower layer free water storages	0.0-0.8
12	*PCTIM*	Permanent impervious area fraction	
13	*ADIMP*	Maximum fraction of an additional impervious area due to saturation	
14	*RIVA*	Riparian vegetation area fraction	
15	*SIDE*	Ratio of deep percolation from lower layer free water storages	
16	*RSERV*	Fraction of lower layer free water not transferrable to lower layer tension water	

2.1, the SAC-SMA model is a typical storage type model that assumes that all rainfall losses are allocated in the upper and lower storages of a conceptual soil profile. Each layer consists of fast components (free water) driven mostly by gravitational forces, and slow components (tension water) driven by an evapotranspiration and diffusion. According the soil moisture property definition, Koren et al., [2000] assumed that slow component storages of the SAC-SMA model are related to available soil water, and that fast component storages are related to gravitational soil water. Available soil water and gravitational soil water were derived from soil properties such as the saturated moisture content, θ_s, field capacity, θ_{fld}, and wilting point, θ_{wlt}. These soil properties can be estimated from STATSGO dominant soil texture grids available for eleven soil layers (from ground surface to 2.5m beneath) for the conterminous United States [*Miller and White*, 1999]. The combined thickness of the upper and lower layers (as a water depth) was assumed to be equal to the total thickness of gravitational and available water storages to the soil profile depth, Z_{max}. A concept of an initial rain abstraction [*McCuen*, 1982] was used to split the soil profile into the upper and lower layers. The Natural Resources Conservation Service (NRCS) (formerly, Soil Conservation Service (SCS)) developed an approach to estimate the initial rain abstraction based on soil and vegetation type, as well as on soil moisture conditions [*McCuen*, 1982]. In the method by Koren et. al., [2001], it was assumed that under the average soil moisture condition stipulated by NRCS, the upper layer tension water storage is full and the free water storage is empty. In this case, the initial rain abstraction should satisfy the upper layer free water capacity. The upper layer thickness can then be calculated based on a SCS curve number, *CN*, for the soil profile. Under these assumptions all SAC-SMA storages (*UZTWM, UZFWM, LZTWM, LZFSM, LZFPM*) defined in water depth units can be estimated as functions of soil porosity, field capacity, wilting point, soil depth, and SCS curve number [*Koren et al.*, 2000].

A relationship for the depletion coefficient of the lower layer primary free water storage was obtained from the solution of Darcy's equation for an unconfined homogeneous aquifer [*Dingman*, 1993] that required estimation of the saturated hydraulic conductivity, K_s, and the specific yield of soil, μ. The percolation parameter *ZPERC* was estimated from other known SAC-SMA parameters as follows. From Eq. 1, it can be seen that the maximum percolation, I_{max}, occurs when the upper layer is fully saturated and the lower layer is dry:

$$I_{max} = I_o \cdot (1+ZPERC) \tag{3}$$

It, therefore, was assumed that the maximum percolation rate is the maximum contents of the lower layer storages released per time interval Δt. Using these assumptions, an expression for *ZPERC* parameter can be obtained from Eq. 3:

$$ZPERC = \frac{(LZTWM + LZFSM + LZFPM) \,/\, \Delta t - I_o}{I_o} \tag{4}$$

Empirical relationships were suggested for other SAC-SMA parameters, *UZK*, *LZSK*, *REXP*, and *PFREE*. Ratios of field capacity (θ_{fld}/θ_s) and wilting point (θ_{wlt}/θ_s) were used as integrated indexes of soil properties. A few coefficients of these relationships were estimated using calibration results from a number of well calibrated headwater basins. Relationships for the 11 SAC-SMA parameters are presented in Appendix.

3. USE OF *A PRIORI* PARAMETERS FOR ESTIMATING SPATIALLY CONSISTENT PARAMETER SETS FOR HEADWATER BASINS

Limited tests of *a priori* parameters of the SAC-SMA model were presented in Koren et al., [2000] and Duan et al., [2001]. While overall statistics showed that *a priori* parameters compared well to carefully calibrated parameter sets for a few river basins, it was found that these derived relationships could not account for some specific local river basin conditions. Consequently, the accuracy of *a priori* parameters can vary for different regions. As an example, the estimated parameters of the lower layer free water storages may not be reliable in regions with deep ground water because the NRCS soils information is only defined to a depth of 2.5m. The split between the upper and lower layers based on the SCS curve number can also contribute to *a priori* parameter uncertainties. Other limitations arise because the approach is based on physical assumptions regarding relationships between model parameters and soil properties, and between soil properties themselves. Although most assumptions are obvious, some quantitative expressions were assigned empirically using SAC-SMA calibration results from a limited number of river basins. Another limitation of the approach relates to available soil and SCS curve number data. STATSGO data consist of soil texture data derived from 1:250000 scale soil maps and interpolated into 1x1 km grids for 11 soil layers. This introduces some limitations on the reliability of *a priori* parameters due to possible spatial sampling of soil texture over large areas (100-200 km^2 in some regions). Therefore, *a priori* parameters should be adjusted if there are observed rainfall-discharge data. The main objective of these relationships is to give reasonable initial values, and to reduce uncertainties in parameter ranges. Another benefit is that these relationships are based on available physical properties of soils and can be used on ungaged basins.

3.1. Estimation of Soil Based Parameters for Selected River Basins

STASGO dominant soil texture grids [*Miller & White*, 1999] for 11 soil layers were used in this analysis. Hydraulic soil properties θ_s, K_s, and Ψ_s (the saturation matrix potential) and b (the slope of the *Campbell's*, [1974] retention curve) for each USDA texture class were calculated using regression equations from Cosby et al., [1984]:

$$\theta_s = -0.00126\, F_{sand} + 0.489 \tag{5}$$

$$\Psi_s = -7.74 e^{-0.0302 F_{sand}},\ kPa \tag{6}$$

$$b = 0.159\, F_{clay} + 2.91 \tag{7}$$

The percentages of sand, F_{sand}, and clay, F_{clay}, were obtained from midpoint values of each textural class [*Cosby et al.*, 1984] using the USDA textural triangle. Field capacity and wilting point estimates were calculated from the Campbell's matric water potential function using parameter values from Equations 5-7

$$\theta_{fld} = \theta_s\, (\Psi_{fld}/\Psi_s)^{-1/b} \tag{8}$$

$$\theta_{wlt} = \theta_s\, (\Psi_{wlt}/\Psi_s)^{-1/b} \tag{9}$$

Matric potential at the field capacity, Ψ_{fld}, was assumed to be -10 kPa for the 1-3 sandy soil classes (see Table 2), and -20 kPa for all other soil classes [*ASCE*, 1990]. Matric potential at the wilting point, Ψ_{wlt}, was assumed to be - 1500 kPa.

Saturated hydraulic conductivity, K_s, stream channel density, D_s, and specific yield, μ, values for each soil texture class are required to estimate the depletion rate of the lower layer primary free water (see Appendix, Eq. A8). Experimental data [*Li et al.*, 1976] reported by Clapp and Hornberger, [1978] were adopted for the saturated hydraulic conductivity. Stream channel density does not vary much depending on soil properties, and a constant value of 2.5 was assumed in this analysis. Since there are no systematic data of the specific yield of different soils, an empirical relationship was developed for this analysis using limited data reported by Armstrong, [1978]:

$$\mu = 3.5\, (\theta_s - \theta_{fld})^{1.66} \tag{10}$$

Results from Eq. 10 for all soil texture classes and Armstrong's estimates are plotted in Figure 1. A 1.6 value of parameter n (see Appendix, Equations A3, A5-A7, A9) was used to maintain an average ratio between the supplemental and primary storage capacities close to 1/3 [*Koren et al.*, 2000]. The values of physical soil properties used in this analysis are given in Table 2.

The NRCS developed a classification system to estimate a curve number, *CN*, based on soil type, land use, agricultural land treatment class, hydrologic condition, and antecedent soil moisture [*McCuen*, 1982]. To assess these factors, soil surveys and site investigations are recommended in addition to the use of soil-land use maps. Some of the factors could not be assessed in this study because only STATSGO grids were available for analyses. In light of this limitation, we utilized a simplified approach in which curve numbers were estimated based on USDA Hydrologic Soil

Table 2. Physical properties of different soil classes defined for this analysis.

No	Texture class	% sand	% clay	θ_{max}	θ_{fld}	θ_{wlt}	K_s mm/hr	μ
1	Sand	92	3	0.37	0.15	0.04	633.6	0.29
2	Loamy sand	82	6	0.39	0.19	0.05	562.6	0.23
3	Sandy loam	58	10	0.42	0.27	0.09	124.8	0.15
4	Silty loam	17	13	0.47	0.35	0.15	25.9	0.10
5	Silt	9	5	0.48	0.34	0.11	20.0	0.12
6	Loam	43	18	0.44	0.30	0.14	25.0	0.13
7	Sandy clay loam	58	27	0.42	0.29	0.16	22.7	0.12
8	Silty clay loam	10	34	0.48	0.41	0.24	6.1	0.04
9	Clay loam	32	34	0.45	0.36	0.21	8.8	0.07
10	Sandy clay	52	42	0.42	0.33	0.21	7.8	0.07
11	Silty clay	6	47	0.48	0.43	0.28	3.7	0.02
12	Clay	22	58	0.46	0.40	0.28	4.6	0.03

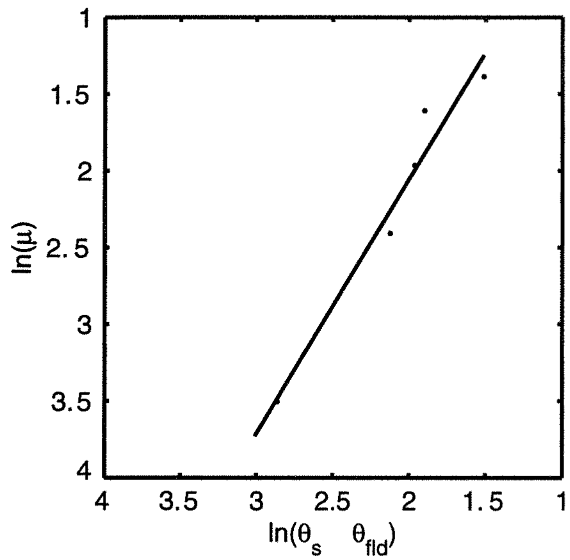

Figure 1. The specific yield (μ) as a function of the free water capacity (θ_s - θ_{fld}). Armstrong's estimates are shown with circles.

Group grids (HSG) [*Miller & White*, 1999] assuming 'pasture or range land use' under 'fair' hydrologic conditions for the entire region [*McCuen*, 1982].

Eleven SAC-SMA parameter grids having a 1x1 km resolution were generated for the conterminous United States using data from Table 2 and HSG-based SCS curve numbers. The lower layer tension water capacity map for the entire USA is displayed in Figure 2 as an example. These grids are now available in an ArcView application called the Calibration Assistance Program (CAP) [*Reed et al.*, 2001] that is designed to assist the calibrator in deriving initial parameter estimates. The CAP computes mean, maximum and minimum values of these parameters for basins and/or elevation zones of interest, and presents the results in a tabular format.

3.2. Tests Design and Data

Two tests, model parameter transferability to ungaged basins and constrained automatic calibration, were performed for a number of headwater watersheds in the Ohio river basin, Table 3. Rainfall-runoff simulations were generated in a lumped mode assuming that input data and model parameters were uniform over each basin. *A priori* SAC-SMA parameters for each watershed were estimated as an arithmetic averages from 1x1 km resolution parameter grids generated as described in Section 3.1.

3.2.1. Parameter transferability test. First, a test of SAC-SMA parameter transferability is performed on a number of neighboring headwater basins of the Upper Monongahela River, West Virginia (see Figure 3, watershed numbers 1-6). These watersheds are located in the southeastern portion of the Upper Monongahela basin. Slight differences in mean basin elevation exist. Comparison of observed hydrographs

Figure 2. The lower layer tension water capacity derived from soil data for the conterminous US.

Table 3. List of basins selected for the analysis.

No.	Watershed name	Latitude	Longitude	Basin area, ml^2	Elevation, ft.
		First group of basins			
1	Dry Fork at Hendricks, WV	39.072	-79.623	349.0	3240
2	Buckhannon R. at Hall, WV	39.051	-80.115	277.0	2060
3	Middle Fork R. at Audra, WV	39.040	-80.068	148.0	2850
4	Blackwater R. at Davis, WV	39.127	-79.469	85.9	3350
5	Tygart Valley R. at Dailey, WV	38.809	-79.882	185.0	2840
6	Shavers Fork below Bowden, WV	38.913	-79.771	151.0	3120
		Second group of basins			
7	Tygart Valley R. at Belington, WV	39.029	-79.936	408.0	1679
8	Middle Island C. at Little, WV	39.475	-80.997	458.0	631
9	Bluestone R. nr Pipestem, WV	37.544	-81.011	394.0	1527
10	Greenbrier R. at Buckeye, WV	38.186	-80.131	540.0	2086
11	Ohio Brush C. nr West Union, OH	38.804	-83.421	387.0	511
12	SF Licking R. at Cynthiana, KY	38.391	-84.303	621.0	689
13	Stillwater R. at Englewood, OH	39.869	-84.282	650.0	700
14	White R. at Noblesville, IN	40.047	-86.017	858.0	738
15	Big Blue R. at Shelbyville, IN	39.529	-85.782	421.0	738
16	Sugar C. nr Edinburgh, IN	39.361	-85.998	474.0	646
17	French Broad R. at Blantyre, NC	35.299	-82.624	296.0	2060
18	French Broad R. at Asheville, NC	35.609	-82.579	945.0	1950

shows much similarity in the response of the watersheds with some variations that are primarily related to elevation. The 'best' SAC-SMA parameter sets for all basins were available from the Ohio River Forecast Center (OHRFC). OHRFC hydrologists used the NWSRFS calibration procedure [*Smith et al.*, this volume] which is based on visual fitting of simulated and observed hydrographs, and comparing different statistics. While this procedure is rather subjective, it provides physically reliable and robust estimates of the SAC-SMA model parameters. Time series of mean areal six-hourly precipitation and air temperature values, and daily discharges were available from the OHRFC. 25-45 year time series were generated for most basins. However, only eight years of historical data were available for the Shavers Fork below Bowden.

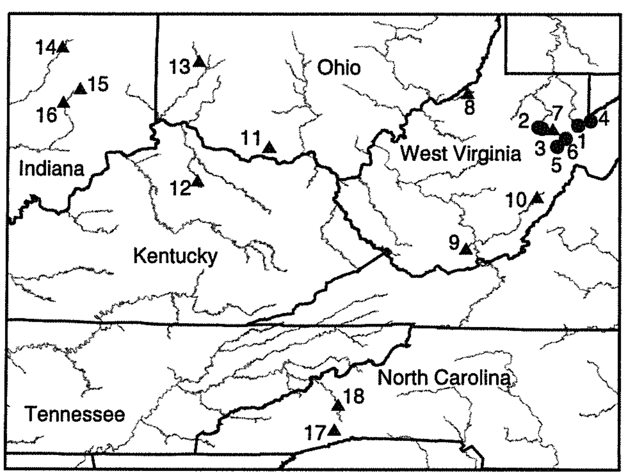

Figure 3. Location of outlets for the first (shown with circles) and second (shown in triangles) groups of test watersheds.

Control simulations for the entire historical period (when input/output data were available) were first performed using two parameter sets for 6 selected watersheds: 1) 'best' OHRFC manually calibrated parameters for each basin, and 2) soil derived parameters for the same basins. These results provide an objective evaluation of the performance of soil derived parameters compared to OHRFC parameters. Because OHRFC parameters were derived using a subjective procedure, a simple comparison of just parameter values can not provide conclusive information.

To test parameter transferability, it was assumed that only one basin had historical time series to perform model calibration. The Dry Fork at Hendricks watershed was selected as representative and the best calibrated for the first group (based on the OHRFC expert judgment [*Tom Adams*, personal communication]). All SAC-SMA parameters for five other watersheds of this group were assumed to be equal to calibrated parameters for the Dry Fork basin, $X_{DFH,calb}$. This approach is usually used when calibration is performed on a large river basin with a number of ungaged watersheds.

Other parameter sets for these watersheds were generated by scaling of soil derived parameters for each basin, $X_{i,soil}$, based on the ratio of calibrated, $X_{DFH,calb}$, and soil derived, $X_{DFH,soil}$, parameters for the Dry Fork basin:

$$X_{i,*} = \frac{X_{DFH,calb.}}{X_{DFH,soil}} X_{i,soil} \qquad (11)$$

where $Xi,_*$ are scaled parameters for a basin j. It assumes that soil information represents reasonably well the spatial pattern of model parameters while their magnitudes may be not optimal.

3.2.2. Use of soil derived parameters in an automatic calibration. This test involves automatic calibration of a larger number of basins representing different climatic and hydrological conditions. The second group of 12 headwater watersheds are spread through the Ohio-Tennessee River basin including Ohio, Indiana, Kentucky, West Virginia, and North Carolina states, see Figure 3, and represent different climatic and hydrological conditions. Annual precipitation varies from 500mm in the northwest portion of the region to 1500mm in the southeastern portion [*Schaake et al.*, 2000]. Potential evaporation varies much less throughout the region. Consequently, significant differences in annual runoff for the northwest (200mm) and southeastern (900mm) portions of the basin are present. Daily precipitation, air temperature, and discharges for 45-50 years period were obtained from the Model Parameter Estimation Experiment (MOPEX) [*Schaake et al.*, 2001] project databases.

First, automatic calibration was performed for all selected basins without the use of soil derived parameters. Parameters were allowed to vary in a broadly defined feasible space [*Brazil*, 1989; *Boyle et al.*, 2001]. Table 1 lists parameter ranges used in this study. The second set of calibration runs were conducted using soil derived parameters to define parameter ranges that are tied to basin physical characteristics. 25 percent bounds from soil derived parameters were used in these runs, i.e.

$$\mathbf{(1\text{-}0.25)}\ X_{i,soil} < X_i < \mathbf{(1\text{+}0.25)}\ X_{i,soil} \qquad (12)$$

The University of Arizona Shuffled Complex Evolution (SCE-UA) calibration technique [*Duan et al.*, 1992] was used in this test. The SCE-UA method is a global search procedure that uses concepts from random search algorithms, along with the strength of the downhill simplex method. It has been tested extensively in the last few years and is found to be efficient and consistent in finding the global optimum of multi-parametric nonlinear problems encountered in the calibration of conceptual watershed models. A weighted error function was selected as a minimization criterion:

$$F = \alpha \cdot MVRMS + (1\text{-}\alpha)\ DRMS \qquad (13)$$

where *MVRMS* is a mean square error of monthly runoff volumes, *DRMS* is a mean square error of daily discharges, and α is a weight parameter; 0.8 value was selected for this test. A 15 year period was used in the calibration process, and the rest of data (usually 25-28 years) were used for validation.

4. RESULTS AND DISCUSSION

4.1. Parameter Transferability Test Results

Some accuracy statistics of hydrographs simulated using calibrated and soil-derived parameters are shown in Table 4. These statistics include a daily discharge root mean square error, *DRMS*, a monthly volume root mean square error, *MVRMS*, a daily discharge root mean square error during flood events only, *FDRMS*, percent of total bias

Table 4. Accuracy statistics of hydrographs simulated using calibrated and soil derived parameters for the Upper Monongahela basin

Basin #	*DRMS* cms	*MRMS*, mm	*BIAS*, %	*FDRMS*, %	R^2
		Calibrated parameters			
1	16.9	17.8	3.2	47	0.86
2	10.2	12.5	1.3	32	0.91
3	6.5	14.1	-0.9	40	0.89
4	4.7	20.8	2.0	49	0.84
5	7.9	12.5	1.6	43	0.89
6	8.1	18.3	-0.6	43	0.87
Avg.	9.1	16.0	1.6 [1]	42	0.88
		Soil derived parameters			
1	18.2	17.6	2.3	50	0.84
2	11.3	12.9	-2.7	40	0.89
3	7.0	14.5	-4.6	40	0.88
4	5.1	20.5	-0.5	47	0.82
5	7.9	12.4	-5.3	44	0.89
6	9.9	18.7	-1.1	51	0.81
Avg.	9.9	16.1	2.7 [1]	45	0.86

1 - Estimated as the average of absolute biases for 6 watersheds.

of simulated and observed hydrographs, *BIAS*, and a correlation coefficient of daily discharges, *R*. As seen in Table 4, calibrated parameters usually produce higher accuracy although the gain is not as significant as compared to use of soil-derived parameters. As an example, simulated and observed hydrographs are plotted in Figure 4 for the Middle Fork River at Audra. Both parameter sets lead to good simulations of the observed hydrographs. The semi-log scale plot in Figure 4b suggests that base flow recessions are not well simulated by the soil-derived parameter sets. A possible reason for this was discussed earlier in Section 3.

Accuracy statistics from the transferability test simulations are shown in Table 5. The values suggest that scaled parameters improved simulation accuracies for 5 'ungaged' watersheds. Furthermore, the accuracy statistics are close to those obtained when each watershed was calibrated independently (compare Tables 5 and 4). While the overall statistics from a single watershed calibration (the Dry Fork at Hendricks watershed) are not greatly different from those derived from the scaled parameter version, there are significant degradations in bias (*BIAS*) and flood (*FDRMS*) statistics for some outlets, specifically for the Middle Fork River at Audra and the Buckhannon River at Hall. The reason for this is that most soil derived parameters do not differ much for selected watersheds excluding the Middle Fork and Buckhannon river basins, see Figure 5, and, as a result, scaled parameters will produce similar results. However, the lower zone tension water (*LZTWM*) and supplemental free water (*LZFSM*) storages as well as the depletion rate of the primary free water storage are much lower for mentioned two watersheds (see Figure 5, thick lines). As a result, scaled parameters produced more runoff for these watersheds, and lead to improved bias and flood statistics compared to the constant parameter case.

4.2. Automatic Calibration Test Results

Calibration and validation results are presented in Figures 6 and 7. Daily runoff errors (*DRMS*) and monthly volume errors (*MVRMS*) from unconstrained and constrained calibration/validation, and soil derived parameter simulations are plotted for 10 watersheds in the Ohio basin. Results from two watersheds in the Tennessee basin were excluded from these plots, and will be discussed later. As seen from Figures 6 and 7, unconstrained calibration leads to slightly better sta-

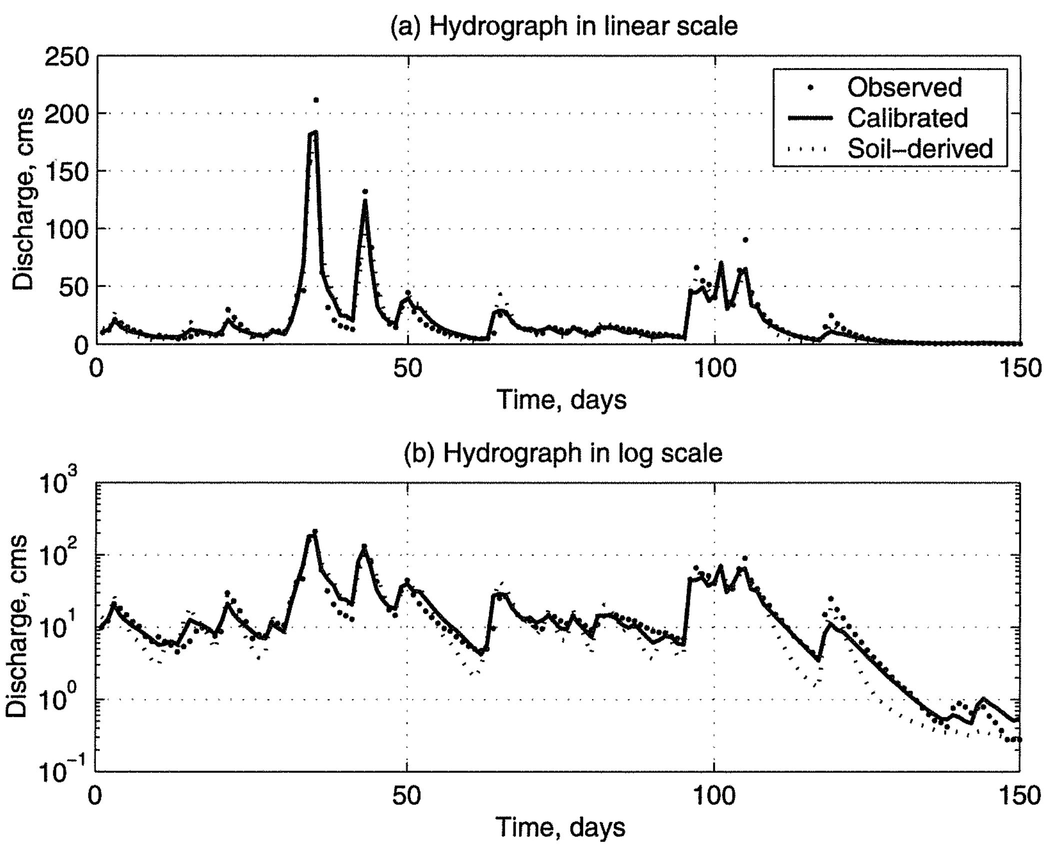

Figure 4. Observed hydrograph and hydrographs simulated using calibrated and soil derived parameters for the Middle Fork River at Audra, February - June 1967.

Table 5. Statistics for the parameter transferability test, the Upper Monongahela river basin.

Basin #	*DRMS*, cms	*MRMS*, mm	*BIAS*, %	*FDRMS*, %	R^2
Parameters calibrated for the Dry Fork at Hendricks (basin #1)					
2	10.8	13.4	-2.0	38	0.90
3	7.2	14.9	-5.3	42	0.87
4	4.6	20.8	1.1	45	0.85
5	7.9	12.2	-3.5	43	0.88
6	8.8	18.6	-0.3	47	0.85
Avg	7.9	16.0	2.4 [1]	43	0.87
'Scaled' soil derived parameters					
2	10.5	13.2	-0.5	36	0.91
3	7.6	14.1	-2.5	34	0.86
4	4.6	21.0	0.4	46	0.85
5	8.1	12.5	-3.5	45	0.88
6	9.0	18.8	-0.1	44	0.85
Avg	8.0	15.9	1.4 [1]	41	0.87

1 - Estimated as the average of absolute biases for 5 watersheds.

tistics compared to constrained calibration on the calibration data sets, however, this gain was practically eliminated on the validation data sets. While the use of soil-derived parameters alone provides reasonable simulation results, minor parameter adjustments can improve the overall performance.

The major benefit of use of soil derived parameters as calibration constraints is in generating spatially consistent parameter sets. The spatial variability of one SAC-SMA parameter, *UZFWM*, derived from unconstrained and constrained (values in parentheses) calibration can be seen in Figure 8. It can be seen that overall, constrained and unconstrained results are consistent for most outlets. However, *UZFWM* values from unconstrained calibration can differ by 3-5 times for neighboring watersheds (highlighted values in italic). Figure 9c shows that the same behavior can be seen for most of the other parameters, which vary over the entire feasible parameter ranges. On the other hand, constrained calibration generates more consistent parameter sets while maintaining hydrological performance as shown in Figure 9b. Comparison of Figures 9a and 9b confirms that the spatial pattern of parameters derived by constrained calibration is consistent with soil derived patterns slightly adjusted to local physical properties and possibly data uncertainties.

Figure 10 shows that the most affected parameters from unconstrained calibration were the percolation parameters *ZPERC* and *REXP*, the upper layer free water storage *UZFWM*, and the lower layer tension water storage *LZTWM*. Deviations of these parameters from soil-based parameters were more than 60%. At the same time, deviations of constrained calibration parameters were much less than the allowed constraints of 25%. Overall constrained calibration results suggest that only 12% of the final parameter values were constrained by the specified search boundaries. Of these, 65% were values of the least identifiable from soil data parameters *LZPK* and *LZTWM*.

5. SUMMARY AND FUTURE WORK

This study illustrates the benefit of using soil-derived parameters to estimate conceptual model parameters for ungaged watersheds, and to improve results of automatic calibration. The results suggest that the use of soil derived parameters can improve the spatial and physical consisten-

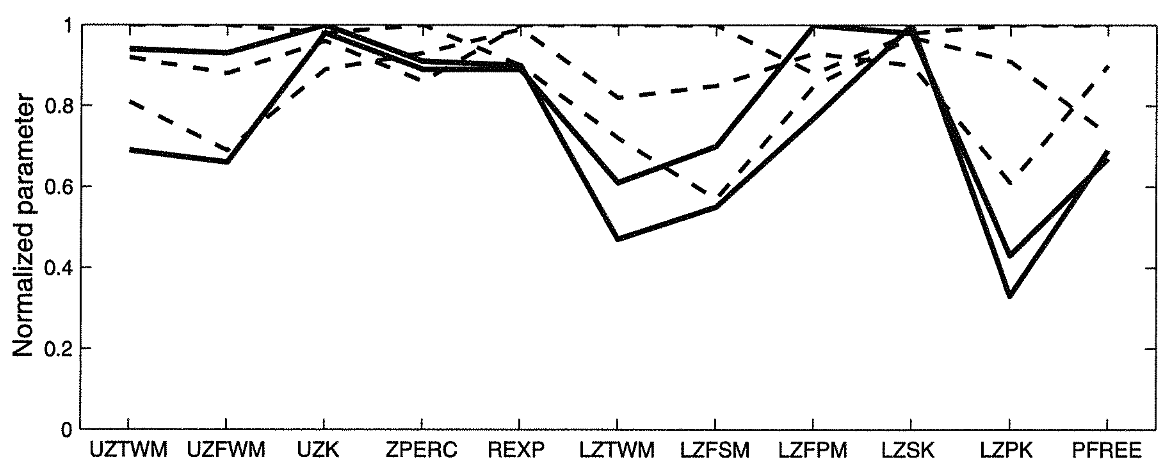

Figure 5. Soil derived normalized model parameters for the transferability test watersheds. The Middle Fork and Buckhannon river watersheds are shown with thick lines. In this Figure and later on, parameters were normalized based on their ranges for the unconstrained optimization.

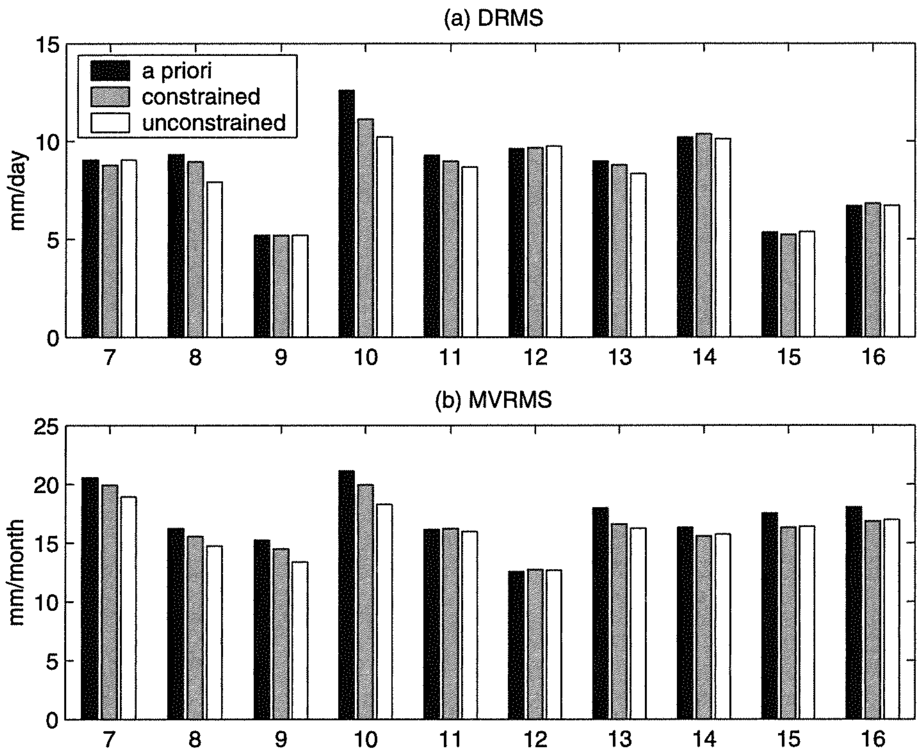

Figure 6. Constrained and unconstrained calibration results for the second group of watersheds: Calibration period.

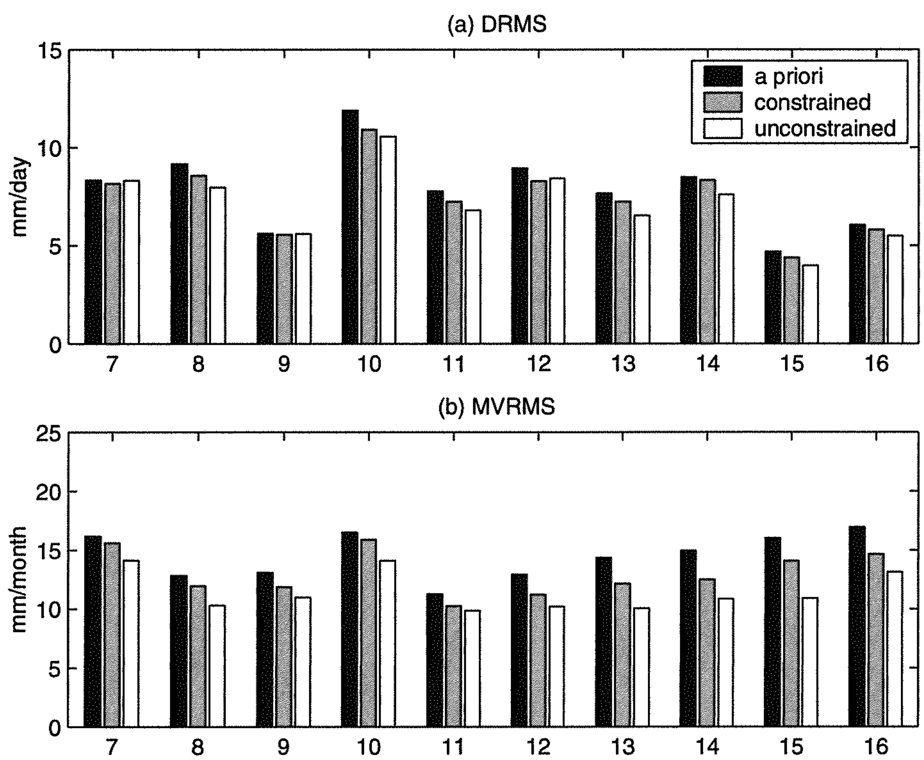

Figure 7. Constrained and unconstrained calibration results for the second group of watersheds: Verification period.

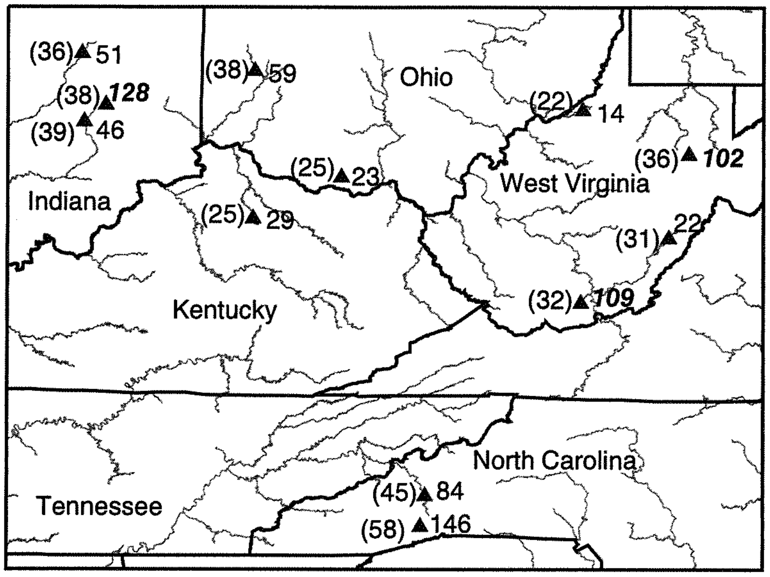

Figure 8. Spatial variability of the upper layer free water capacity derived from unconstrained and constrained (values in parentheses) calibration for the second group of watersheds.

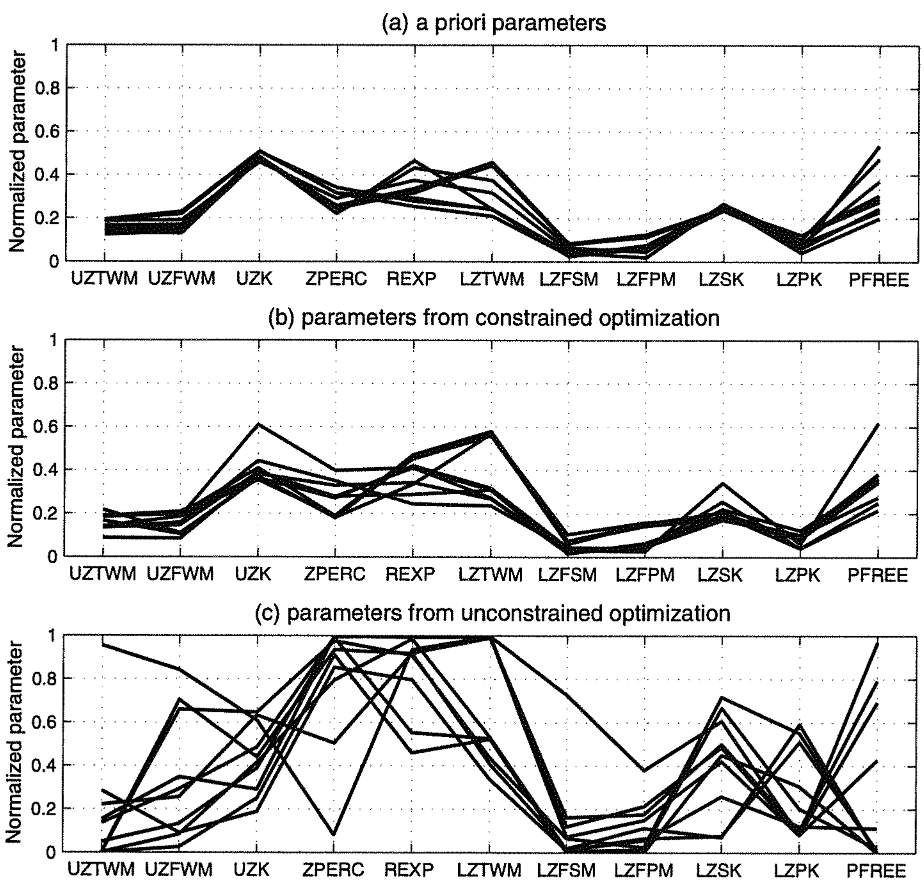

Figure 9. Variation of normalized model parameters obtained from (a) soil data, (b) constrained calibration, and (c) unconstrained calibration for the second group of watersheds.

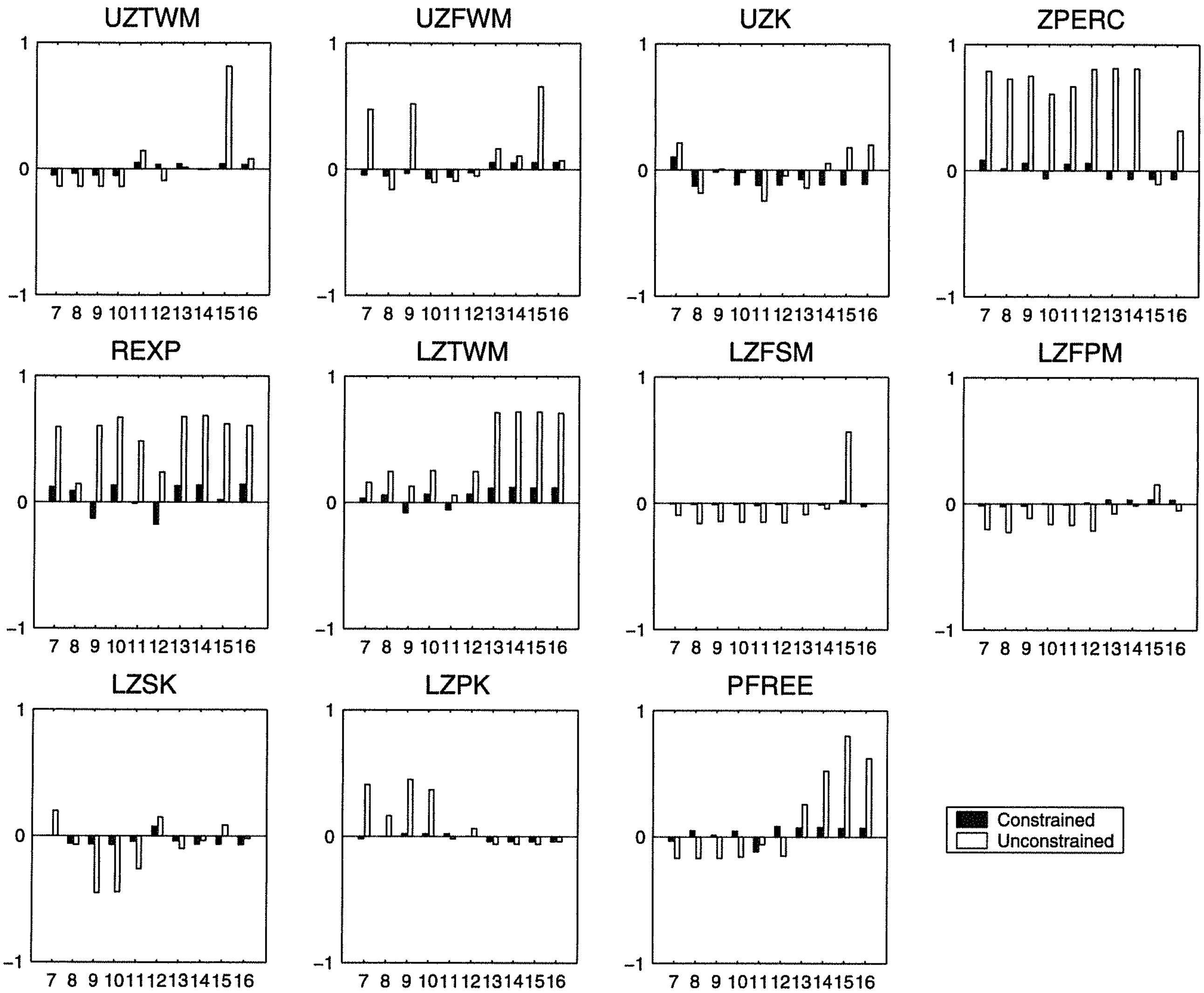

Figure 10. Deviation of constrained and unconstrained normalized parameters from soil derived parameters for the second group of watersheds.

cy of estimated model parameters while maintaining hydrological performance. When transferring model parameters from well-calibrated watersheds to ungaged watersheds, RFC experts rely on qualitative information such as soil, vegetation, etc. Soil derived parameters provide a quantitative measure of possible differences between parameters that allow one to 'rescale' calibrated parameters to ungaged watershed.

Use of constrained calibration reduces non-regularities in the spatial pattern of model parameters. RFC forecasters routinely make run-time adjustments to the hydrologic model states and certain parameters in order to keep the forecast models in close agreement with observed streamflow data. On a practical level, spatially consistent hydrologic model parameters should allow RFC forecasters to more efficiently make these run-time modifications, especially in the case of non-standard conditions during a rainfall event. With spatially consistent parameter sets, the forecaster can expect to use similar adjustments throughout a basin, thus saving time and allowing for the evaluation of more scenarios.

A simple approach was used to incorporate soil derived parameters into the automatic calibration procedure. Search regions of parameters were constrained by some percentage of soil-derived parameters. However, the percentage can vary for different regions depending on the accuracy of soil derived parameters and the quality of input/output data. Large uncertainties of soil derived parameters complicate the calibration procedure and in some cases can eliminate the benefit of using constrained calibration. This problem was encountered when constrained calibration was performed on two watersheds in the Tennessee basin, the French Broad River at Blantyre and Asheville. Daily and monthly statistics from constrained calibration were degraded significantly for both calibration and validation data sets as shown in Table 6. Large uncertainties in the soil derived parameters of the lower layer free water storages are the main reason for this. As discussed above, the soil-based approach does not account for a deep ground water aquifer, and as a result, underestimates the lower layer free water storages. Unconstrained calibration generated much higher values of the lower layer free water parameters for these two basins, *LZFSM*=270mm and *LZFPM*=950mm compared to 25mm and 124mm respectively for the soil derived parameters. To deal with this problem, constrained calibration should account for large uncertainties of soil derived parameters. One possibility would be to use an explicit measure, D_X, of the deviation of calibrated parameters, X_i, from soil derived parameters, X_i^*, in the automatic calibration procedure:

$$D_x = \left(\sum_{i=1}^{N} \left(\frac{X_i^* - X_i}{X_{\max,i} - X_{\min,i}} \right)^2 \right)^{0.5} \qquad (14)$$

where $X_{max,i}$ and $X_{min,i}$ are maximum and minimum parameter values in a feasible space, and N is the number of calibrated parameters. A single objective function can be selected that will weight the gain in simulation accuracy versus the increase in D_X. However, estimation of a weight function may be a real challenge of this approach. Another possibility would be to incorporate a parameter deviation measure into multi-criteria calibration [*Boyle et al.*, 2000].

As stated in Section 3, there are weaknesses in the derivation of soil-based SAC-SMA parameters. Future research should be conducted to include more data sources in the estimation technique. Specifically, ground water information and hydrograph analysis may be helpful in estimating lower layer free water storages and depletion coefficients. New developments in generating more consistent SCS curve number grids can also lead to better estimates of fast runoff parameters.

APPENDIX: SOIL BASED RELATIONSHIPS FOR ESTIMATING A PRIORI PARAMETERS OF THE SAC-SMA MODEL

Below there are SAC-SMA parameter and soil property relationships as they appeared in Koren et al., [2000]. Two printing errors in the original paper were fixed here: a coefficient 4 in the denominator was removed and a basic time step, Δt, (in the SAC-SMA model it equals 24 hours) was added in Eq. A8, and a coefficient 50.8 in Eq. A12 was replaced by 5.08. Parameter and soil property notations are consistent with Table 1 and Section 3.

Upper layer parameters:

$$UZTWM = (\theta_{fld} - \theta_{wlt}) \cdot Z_{up} \qquad (A1)$$

$$UZFWM = (\theta_s - \theta_{fld}) \cdot Z_{up} \qquad (A2)$$

$$UZK = 1 - (\theta_{fld}/\theta_s)^n \qquad (A3)$$

Lower layer parameters:

$$LZTWM = (\theta_{fld} - \theta_{wlt}) \cdot (Z_{max} - Z_{up}) \qquad (A4)$$

$$LZFSM = (\theta_s - \theta_{fld}) \cdot (Z_{max} - Z_{up}) \cdot (\theta_{wlt}/\theta_s)^n \qquad (A5)$$

$$LZFPM = (\theta_s - \theta_{fld}) \cdot (Z_{max} - Z_{up}) \cdot [1-(\theta_{wlt}/\theta_s)^n] \qquad (A6)$$

$$LZSK = \frac{1-(\theta_{fld}/\theta_s)^n}{1+2(1-\theta_{wlt})} \qquad (A7)$$

Note: Equation A8 has been revised. See Addendum

Table 6. Daily, DRMS, and monthly, MVRMS, statistics for two watersheds of the French Broad river from automatic calibration test

Basin #	*DRMS*, cms Uncon-strained	Con-strained	Soil derived	*MVRMS*, mm Uncon-strained	Con-strained	Soil derived
			Calibration period			
17	4.35	7.39	13.61	16.73	31.66	36.95
18	10.79	15.39	24.67	10.88	21.97	26.37
			Validation period			
17	4.08	5.65	10.30	18.62	31.11	38.21
18	10.20	13.47	20.75	9.73	19.47	25.11

$$LZPK = 1 - e^{-\frac{\pi K_s D_s (Z_{max} - Z_{up})\Delta t}{\mu}} \quad \text{(A8)}$$

$$PFREE = (\theta_{wlt}/\theta_s)^n \quad \text{(A9)}$$

$$ZPERC = \frac{LZTWM + LZFSM \cdot (1 - LZSK)}{LZFSM \cdot LZSK + LZFPM \cdot LZPK} + \frac{LZFPM \cdot (1 - LZPK)}{LZFSM \cdot LZSK + LZFPM \cdot LZPK} \quad \text{(A10)}$$

Percolation parameters:

$$REXP = [\theta_{wlt}/(\theta_{wlt,sand} - 0.001)]^{0.5} \quad \text{(A11)}$$

Upper layer thickness:

$$Z_{up} = 5.08 \frac{1000/CN - 10}{\theta_s - \theta_{fld}}, \; mm \quad \text{(A12)}$$

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Addendum
December, 2011

New version of equation (A8)

Equation (A8) is corrected to account for the total ground water thickness (H). Equation (A8) is derived from a solution of the ground water recession:

$$Q_t = Q_o \cdot e^{-\alpha \cdot t} \quad \text{(A8.1)}$$

where

$$\alpha = \frac{\pi^2 K_s H}{4 \cdot \mu \cdot l^2} \quad \text{(A8.2)}$$

where K_s is the saturated hydraulic conductivity, μ is the specific yield of a ground water layer, l is the hillslope length that can be derived from the drainage density assuming a rectangle hillslope, $l = 1/(2D_s)$. Substitution this relationship in (A8.2) leads to:

$$\alpha = \frac{\pi^2 K_s D_s^2}{\mu} H \quad \text{(A8.3)}$$

In equation (A8), thickness of the SAC-SMA lower zone ($Z_{max} - Z_{up}$) is used as a ground water thickness. However, the lower zone thickness of SAC-SMA reflects only dynamical part of the ground water layer. Therefore, to account for the total ground water thickness, a very slow component of the ground water layer, H_o, should be added:

$$H = (Z_{\max} - Z_{up}) + H_o \quad \text{(A8.4)}$$

Representing H_o as a multiple of the lower zone thickness, one can define a total thickness as:

$$H = (1 + \beta) \cdot (Z_{\max} - Z_{up}) \quad \text{(A8.5)}$$

and a final form of the corrected equation (A8):

$$LZPK = 1 - \exp\left[-\frac{\pi^2 K_s D_s^2 (1+\beta) \cdot (Z_{\max} - Z_{up}) \cdot \Delta t}{\mu} \right] \quad \text{(A8.6)}$$

Because often there were no comprehensive aquifer thickness data, the constant value of β=16 was used in all our simulations. This value agrees with an average ground water thickness of USA aquifer (Maxey, 1964, Table 4-1-2) of about 15 m.

Reference:

Maxey, G.B., 1964. In Book: Handbook of applied hydrology (Ed. Ven te Chow), McGraw-Hill Book Company, Chapter 4-1, Hydrogeology.